



Analytical model for predicting the effect of operating speed on shaft power output of Stirling engines

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ABSTRACT

This paper is concerned with numerical predictions of relationship between operating speed and shaft power output of Stirling engines. Temperature variations in expansion and compression spaces as well as the shaft power output corresponding to different operating speeds were investigated by using a lumped-mass transient model. Effects of major operating parameters on power output were studied. Results show that as the operating speed increased, temperature difference between the expansion and compression spaces was reduced and as a result, the shaft work output decreased. However, the shaft power output is determined in terms of the shaft work output and the operating speed. When the operating speed was elevated, the shaft power output reached a maximum at a critical operating speed. Over the critical operating speed, the shaft power output decreased in high-speed regime. In addition, as air mass was reduced, either a decrease in thermal resistances or an increase in effectivenesses of the regenerator leads to an increase in the engine power.

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1. Introduction

Owing to rapid depletion of fossil fuels and threat of global warming, environmental problems have received great attention in recent years. A large number of researchers concentrate their focus on the electricity generation from solar energy which is treated as one of potential energy sources in near future. Practically, there are two major effects to convert the solar energy into electricity: photovoltaic and thermodynamic. The photovoltaic (PV) effect is used to produce electricity when sunlight hits a PV panel and releases electrons from special layers of silicon and pushes them across an electric field. On the other hand, the thermodynamic effect is used in devices like concentrating solar power systems with Stirling engines (CSP-Stirling) [1]. The CSP-Stirling system is typically equipped with a solar energy receiver to absorb the solar radiation and the Stirling engine to convert the absorbed solar energy to mechanical energy and then electricity [2,3]. In addition, the Stirling engines can also be applied in automobiles and submarines as power resources [4,5].

The Stirling engines are referred to as external combustion heat engines that are operated based on a regenerative closed power cycle using air, nitrogen, helium, or hydrogen as working fluid [6]. In theory, the thermal efficiency of an ideal regenerative Stirling cycle

is equal to the Carnot cycle efficiency under the same thermal reservoirs temperatures. As compared with other alternative power generation methods, the CSP-Stirling system features high efficiency and modular construction. Therefore, cost of these systems is expected to become more competitive in the distributed markets in near future. However, it is noted that performance of the CSP-Stirling system for generating electrical power is dependent on energy conversion and mechanical efficiencies of the Stirling engine. Therefore, predictions and improvement in the performance of the Stirling engine are of major concerns to related investigators.

An ideal regenerative Stirling cycle consists of four processes in a cycle. Firstly, working fluid absorbs heat from high temperature reservoir and experiences an isothermal expansion (1→2). Secondly, the hot working fluid flows through a regenerator, and the regenerator absorbs heat from the hot working fluid. Thus, temperature of the working fluid decreases in an isochoric process (2→3). Thirdly, the working fluid rejects heat to a low temperature reservoir and experiences an isothermal compression (3→4). Finally, the cold working fluid flows back through the regenerator, and the regenerator rejects heat to the working fluid. The temperature of work fluid increases in the second isochoric process (4→1). The *p*-*V* diagram of the ideal Stirling cycle (1→2→3→4→1) is plotted in Fig. 1.

As mentioned in Ref. [7], the Stirling engine was invented by Robert Stirling in 1816, which was called 'hot-air engine' originally, and the first successful reversible model for analysis of the Stirling

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Nomenclature

C_p	specific heat (J/kg·K)
E	mechanism effectiveness
e_C	effectiveness of regenerator in cooling process
e_H	effectiveness of regenerator in heating process
L_r	length of regenerator (m)
M	molecular weight of working fluid (kg/mol)
m	mass of working fluid (kg)
N	number of moles
P_s	shaft power output (W)
p	pressure of work space (N/m ²)
Δp	pressure difference between expansion and compression spaces (N/m ²)
$Q_{\text{gen. absorb}}$	heat absorbed by regenerator from working fluid (J)
$Q_{\text{gen. reject}}$	heat rejected by regenerator to working fluid (J)
R	gas constant (J/K·mol)
R_C	thermal resistance of cold side (K/W)
R_H	thermal resistance of hot side (K/W)
r_h	hydraulic radius of regenerator (m)
T_C	cold side reservoir temperature (K)
T_{Rm}	mean fluid temperature in regenerator channel (K)
T_H	hot side reservoir temperature (K)
T_L	fluid temperature in compression space (K)
T_{Lm}	mean fluid temperature in compression space (K)
T_U	fluid temperature in expansion space (K)
T_{Um}	mean temperature in expansion space (K)
T'_L	initial temperature of cooling process (K)
T'_U	initial temperature of heating process (K)

t	time (sec)
U	gas velocity through regenerator (m/s)
V_c	volume of compression chamber (m ³)
V_d	dead volume (m ³)
V_e	volume of expansion chamber (m ³)
V_r	volume of regenerator (m ³)
V	total volume (m ³)
W	indicated work (J)
W_s	shaft work (J)
W_-	forced work (J)

Greek symbols

θ	crank angle (deg)
ρ	density of working fluid (kg/m ³)
τ	operating temperature ratio
ω	operating speed (rad/s)
ω_o	operating speed (rpm)

Subscripts

C	cold side
c	compression space
d	dead space
e	expansion space
ex	experimental
th	theoretical
H	hot side
m	mean
r	regenerator

engine was presented by Schmidt [8] in 1871. In the reversible model, a closed-form solution for indicated work output of the engine was obtained by assuming that the pressure of the working fluid is uniform throughout the working space of the engine and that the variation in the pressure with time follows sinusoidal function. In the past several decades, the analysis has been extended to irreversible models. For examples, Blank and Wu [9] optimized the power of solar-radiant Stirling engine by using finite-time analysis. Erbay and Yavuz [10] used polytropic processes

to replace the isothermal processes and introduced a realistic regenerator into the model. Wu et al. [11] also studied the irreversible model of the Stirling engine with an imperfect regenerator. The authors presented a criterion for optimizing the dead volume and studied the relationship between net power output and thermal efficiency. Kaushik and Kumar [12,13] performed a finite time thermodynamic analysis of the power output and the thermal efficiency for the Stirling engine with internal heat loss in the regenerator and with finite-heat-capacity external reservoirs. Senft [14,15] took into account internal heat losses and mechanical friction losses and determined theoretical limits on the performance of Stirling engines as well as the maximum value of the mechanical efficiency. Petrescu et al. [16] used a direct method to investigate the irreversible Stirling cycle model with finite speed. In this report, effects of the regenerator loss on the power output and the thermal efficiency of the engines were evaluated. Recently, Kongtragool and Wongwises [17] extended the thermodynamic model to include the dead volumes inside the working spaces. Qin et al. [18] used a linear phenomenological heat-transfer law to analyze an 'endoreversible' Carnot heat engine. The authors found that the power output, the thermal efficiency and the heat-conduction losses are dependent on frequency and heat transfer time ratio. Karabulut et al. [19] compared different kinds of β -type Stirling engines and studied the effect of convective heat transfer coefficient on the power output at different operating speeds. Meanwhile, experiments were conducted by Kongtragool and Wongwises [20] for the low temperature differential γ -type Stirling engines. In this study, engine torque, shaft work, and thermal efficiency of the engines were measured at different engine speeds.

Most recently, Cheng and Yu [21] proposed an irreversible thermodynamic model for the β -type Stirling engines. Periodic variation of pressures, volumes, temperatures, masses, and heat transfers in expansion and compression chambers of the engines

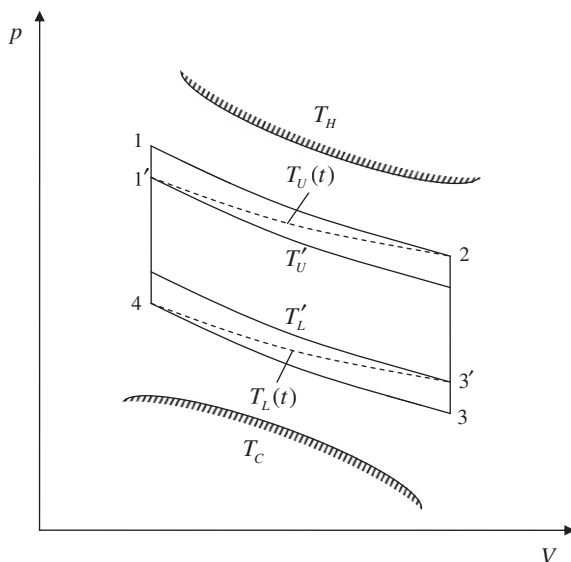


Fig. 1. p - V diagram of Stirling cycle with imperfect regenerator.

are predicted by taking into account heat transfer effectiveness of regenerative channels as well as thermal resistances of heating and cooling heads. Lately, Cheng and Yu [22] first performed a dynamic simulation of the β -type Stirling engines with a cam-drive mechanism by combining dynamic and thermodynamic models. Transient variation in rotational speed of the engines during start-up period can then be predicted.

Performance curves showing the shaft power output at different operating speeds of the Stirling engines may be obtained by analytical or experimental methods based on the above literature survey. However, unfortunately, in spite of the valuable analytical and experimental studies performed in Refs. [19,20,23], so far only a limited number of analytical models were presented that are capable of predicting the dependence of the shaft power output on the operating speed. In consequence, the effects of operating parameters such as the operating temperature ratio, the mass of working fluid, the thermal resistances of the heater and the cooler, the effectiveness of the regenerator, and the properties of working fluids on the performance curves of the Stirling engines are not sufficiently understood.

Under these circumstances, this study is aimed to develop an analytical model that can be used to yield the relationship between the shaft power output and the operating speed of the Stirling engines with imperfect regenerators. The analytical model includes a lumped-mass concept such that transient variation of the thermodynamic properties of the working fluid in each process of the cycle can be predicted. This approach first allows the effects of the operating parameters on the performance curve to be evaluated.

2. Analytical model

Let T_H and T_C represent the temperatures of the hot side and the cold-side reservoirs, respectively, and T_U and T_L represent the fluid temperatures in the expansion and the compression spaces, respectively. Note that $T_U < T_H$ and $T_L > T_C$. Heat transfer between the working fluid and the thermal reservoirs is strongly dependent on the temperature differences, $T_H - T_U$ and $T_L - T_C$, and time for heat transfer. Furthermore, because the time for heating in the expansion space and that for cooling in the compression space in a cycle are decreased as the operating engine speed is increased; therefore, T_U is reduced and T_L is elevated by increasing the operating speed. As a result, the differences $T_H - T_U$ and $T_L - T_C$ both increase with the operating engine speed. It is noticed that in spite of the constant thermal reservoirs temperatures, the working fluid temperatures in the working spaces are varied with time in the respective processes, which may be denoted by $T_U(t)$ and $T_L(t)$.

Temperature changes in the processes of the cycle must be examined. Notice that due to the imperfection of the regenerator, the working fluid enters the expansion space at T'_U , not T_U , at state 1' after passing through the regenerator. Meanwhile, it enters the compression space at T'_L , not T_L , at state 3' after flowing back through the regenerator. The real cycle with an imperfect regenerator is illustrated in Fig. 1.

In this study, a lumped-mass transient model is developed. The working fluid of mass m is regarded as a lumped mass that travels through four working spaces (expansion space, regenerator channels, compression space, and regenerator channels) and experiences changes in temperature and pressure in the cycle. The temperature changes of the working fluid in the four working spaces in the cycle are indicated in Fig. 2.

2.1. Expansion space

In the expansion space, rate of heat absorbed by the working fluid from the hot side reservoir is expressed as $\dot{q}_{in} = (T_H - T_U)/R_H$,

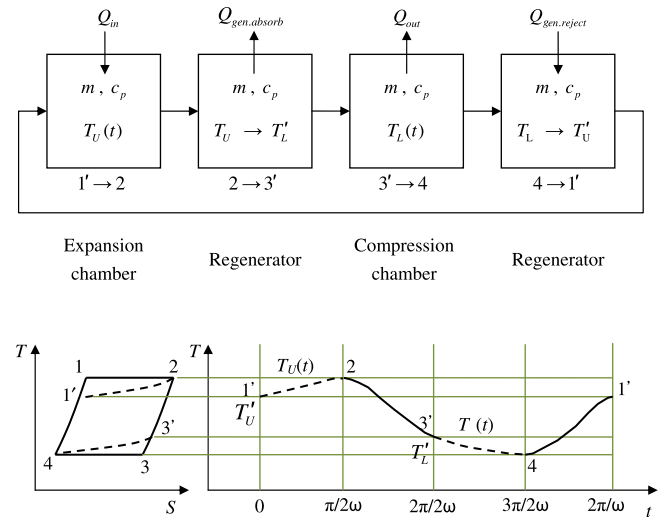


Fig. 2. Temperature changes of lumped mass in the processes.

where $\dot{q}_{in}$ denotes the rate of heat absorbed by the working fluid, and R_H is the thermal resistance of the hot side. Magnitude of R_H is dependent on the operating speed, properties of working fluid, operating temperature, design of heater, and the heating method, which can be determined by experiments. To simplify the analysis, here the value of R_H is fixed. Based on the energy conservation law, $\dot{q}_{in}$ is approximately equal to rate of change in enthalpy of the working fluid. That is,

$$mC_p \frac{dT_U}{dt} = \frac{(T_H - T_U)}{R_H} \quad (1)$$

where m is the mass of the working fluid, which is the product of the molecular weight (M) and the molar number of working fluid (N) contained in the working space. Eq. (1) is a first-order ordinary differential equation. Thus, one can carry out the solution of the equation as

$$T_U(t) = T_H - C_1 \exp\left(-\frac{t}{mC_p R_H}\right) \quad (2)$$

where C_1 is a constant to determine. The working fluid enters the expansion space at T'_U and there it is heated in process 1'-2. Therefore, introducing the initial condition $T_U(0) = T'_U$ yields $C_1 = T_H - T'_U$. The solution can be rewritten as

$$T_U(t) = T_H - (T_H - T'_U) \exp\left(-\frac{t}{mC_p R_H}\right) \quad (3)$$

2.2. Compression space

Similarly, in the compression space, rate of heat rejected by the working fluid to the cold side reservoir is expressed as $\dot{q}_{out} = (T_L - T_C)/R_C$, where $\dot{q}_{out}$ is the heat rejected by the working fluid, and R_C is the thermal resistance of the cold side. The energy conservation law gives

$$mC_p \frac{dT_L}{dt} = -\frac{(T_L - T_C)}{R_C} \quad (4)$$

Using the initial condition $T_L(0) = T'_L$, one has the solution for the temperature of compression space

$$T_L(t) = T_C + (T'_L - T_C) \exp\left(-\frac{t}{mC_p R_C}\right) \quad (5)$$

2.3. Regenerator

The effectivenesses of regenerator can be defined as

$$e_H = \frac{T'_U - T_L}{T_U - T_L} \quad \text{for process } 4-1', \quad (6a)$$

and

$$e_C = \frac{T_U - T'_L}{T_U - T_L} \quad \text{for process } 2-3' \quad (6b)$$

Magnitudes of the regenerator effectivenesses are dependent on porosity, permeability and material of porous matrix. Temperature distribution in the regenerator can be measured by experiments [7,24–26] to determine the regenerator effectivenesses. Furthermore, the magnitudes of the regenerator effectivenesses are also dependent on the operating speed. Actually, they decrease with an increasing operating speed [27,28]. This is because the time for heat exchange between the gas and the porous matrix of the regenerator is decreased by increasing the operating speed. The relation between the regenerator effectiveness and the operating speed is essential and worthy of further investigation. Note that with the imperfect regenerators, one has $e_H < 1$ and $e_C < 1$. Heat transfer from the regenerator to the working fluid of mass m moving through the regenerator channels in the direction from the compression space to the expansion space (in process 4-1') is $Q_{\text{gen reject}} = mC_p(T'_U - T_L)$, and the heat transfer to the regenerator from the working fluid moving from the expansion space to the compression space (in process 2-3') is $Q_{\text{gen absorb}} = mC_p(T_U - T'_L)$.

2.4. Mean temperatures in working spaces

2.4.1. From Eqs. (6a) and (6b), T'_U and T'_L can be calculated by

$$T'_U = T_L + e_H(T_U - T_L) \quad (7a)$$

and

$$T'_L = T_U - e_C(T_U - T_L) \quad (7b)$$

Substituting Eqs. (7a) and (7b) into Eqs. (3) and (5) yields two simultaneous algebraic equations for $T_U(t)$ and $T_L(t)$. The solutions can then be carried out as

$$T_U(t) = \Delta_U / \Delta \quad (8a)$$

and

$$T_L(t) = \Delta_L / \Delta \quad (8b)$$

where

$$\Delta = (1 - e_H)(1 - e_C) - [\exp(t/nWc_p R_H) - e_H][\exp(t/nWc_p R_C) - e_C] \quad (8c)$$

$$\Delta_U = -[\exp(t/nWc_p R_H) - 1][\exp(t/nWc_p R_C) - e_C]T_H - (1 - e_H)[\exp(t/nWc_p R_C) - 1]T_C \quad (8d)$$

$$\Delta_L = -(1 - e_C)[\exp(t/nWc_p R_H) - 1]T_H - [\exp(t/nWc_p R_H) - e_H][\exp(t/nWc_p R_C) - 1]T_C \quad (8e)$$

In practices, the times consumed by the four processes are just slightly different, and hence, the time each process needs is approximated as a quarter of a cycle period. If the engine's operating speed is ω , in rad/s, the time required in each process will be $\pi/2\omega$ [$= (1/4)(2\pi/\omega)$]. Let T_{Um} and T_{Lm} be mean temperatures of the working fluid in the expansion space and the compression space, respectively, which are calculated by

$$T_{Um} = \frac{1}{\pi/2\omega} \int_0^{\pi/2\omega} T_U(t) dt = T_{Um}(\omega) \quad (9a)$$

$$T_{Lm} = \frac{1}{\pi/2\omega} \int_0^{\pi/2\omega} T_L(t) dt = T_{Lm}(\omega) \quad (9b)$$

As suggested in [8,29], the operating temperature ratio is defined by

$$\tau = \frac{T_{Lm}(\omega)}{T_{Um}(\omega)} \quad (10)$$

The mean temperature of the working fluid contained in the regenerator channels can be evaluated as suggested in [17,29] with

$$T_{Rm}(\omega) = \frac{T_{Um}(\omega) + T_{Lm}(\omega)}{2} \quad (11)$$

2.5. Power output

Before calculation of the power output of the engine, one needs to evaluate pressure difference between the expansion and the compression spaces. In terms of gas velocity flowing through the regenerator, diameter of the regenerative channel, and kinematic viscosity of the working fluid at the average operating temperature, the magnitude of Reynolds number is estimated to be 10^2 approximately. According to the experimental data presented by Kays and London [28], at this Reynolds number the magnitude of friction factor is about 10^{-1} . The friction factor is defined as

$$f = \frac{r_h}{L_r} \frac{\Delta p}{\rho U^2 / 2} \quad (12)$$

where r_h and L_r are hydraulic radius and length of the regenerative channels, respectively, ρ is density of the working fluid, U is the gas flow velocity through the regenerator, and Δp is the pressure difference between the expansion and the compression spaces. Magnitude of Δp can then be evaluated as

$$\frac{\Delta p}{p_m} = \frac{L_r U^2 f}{2 r_h R T_C} \sim \frac{10^{-1} \times 10^1 \times 10^{-1}}{10^{-1} \times 10^1 \times 10^2} \sim 10^{-3} \quad (13)$$

Thus, the pressure difference between the expansion and the compression spaces can be negligible. The pressure is assumed to be uniform throughout the engine and can be calculated in terms of the volumes and their mean temperatures of the working spaces [17,21,22,29] as

$$p = \frac{mR}{\frac{V_e}{T_{Um}} + \frac{V_c}{T_{Lm}} + \frac{V_r}{T_{Rm}}} \quad (14)$$

where R is gas constant; and V_e , V_c , and V_r represent instantaneous volumes of the expansion space, the compression space, and the

regenerator channels, respectively. In the derivation of the above equation, the working fluid is assumed to be an ideal gas. Total volume of the engine is $V = V_e + V_c + V_r$ and dead volume of the engine is $V_d = V_{emin} + V_{cmin} + V_r$, where V_{emin} and V_{cmin} are clearance volumes of the expansion and the compression spaces, respectively.

The function $p(V)$ is affected by the operating speed of engine (ω). Using Eq. (14), one can plot the p - V diagram at specific operating speed. Then, indicated work (W) and forced work (W_-) of a cycle can be further calculated. The definitions of the indicated work and the forced work are already described in [14,15,29], which are illustrated in Fig. 3. The indicated work is calculated by integrating the area inside the cycle in the p - V diagram. That is,

$$W = \int_{\text{cycle}} p dV \quad (15)$$

On the other hand, the forced work is represented by the shaded areas shown in the plot. p_b is back pressure exerted on opposite side of the piston. The back pressure could usually be atmospheric pressure or an elevated pressure in case a pressurized space is placed on the opposite side as a buffer zone.

The shaft work of engine can be determined in terms of the indicated work and the forced work, as suggested by Senft [15,29] as

$$W_s = EW - \left(\frac{1}{E} - E\right)W_- \quad (16)$$

where E is referred to as mechanism effectiveness. The mechanism effectiveness is well known as a ratio measure of how well a mechanism utilizes work if it is regarded as a work transmitter. It is a ratio of the work it transfers out through an actuator or puts into internal storage to the work it receives through another actuator or takes from its internal source. Typical value of the mechanical effectiveness is in the range of $0.7 < E < 0.8$. In this study, the mechanism effectiveness is assigned to be 0.7. Note that effects of friction are included in the mechanism effectiveness. Detailed information for the mechanism effectiveness has been provided by Senft [15,29]; therefore, no further description is provided here to save space. The shaft

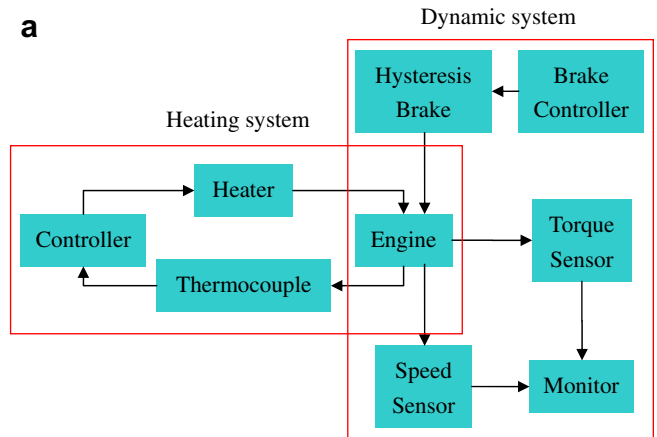
power output can then be calculated from the obtained shaft work with

$$P_s = W_s(\omega/2\pi), \text{ in Watt} \quad (17)$$

In the present study, air is used as the working fluid. For air, the molecular weight is $M = 2.897 \times 10^{-2}$ kg/mol and the specific heat $c_p = 1087$ J/(kg·K). Meanwhile, the hot side and the cold-side reservoir temperatures, T_H and T_C , are fixed at 1200 K and 300 K, respectively.

2.6. Experimental apparatus and testing procedure

The purpose of the experiment is to measure the shaft power output of engine under different operating speed for validation. Experimental apparatus is shown in Fig. 4. A non-contact infrared heater was used for heating the engine, and surface temperatures at the hot and the cold sides are measured by using K-type thermocouples. In order to maintain constant heating temperature (T_H), a PID controller was used to control the temperature by adjusting the electrical voltage supplied to the infrared heater. The engine is an α -type Stirling engine with Ross mechanism, of which the specifications are given in Tables 1 and 2. The engine shaft was



Schematic of the performance test system

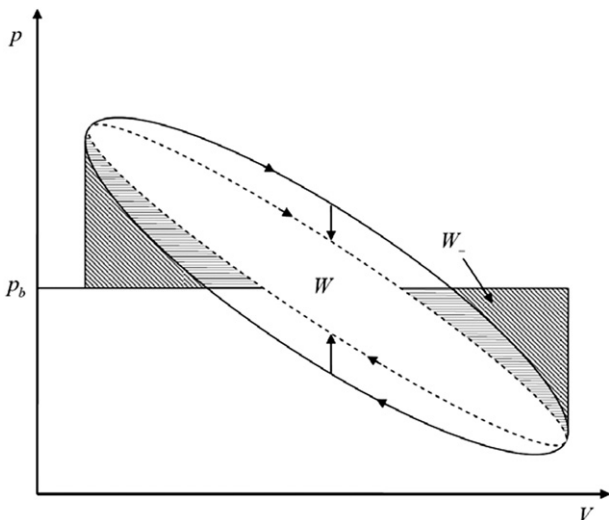


Fig. 3. Definitions of indicated work (W) and forced work (W_-). If the area of cycle is reduced, W is decreased while W_- is increased.

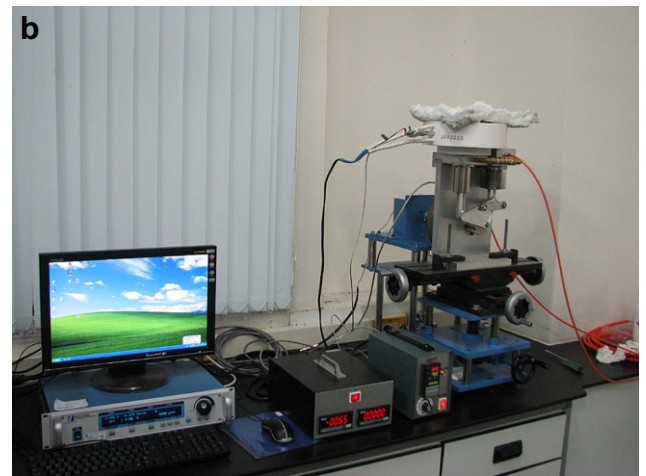


Photo of the performance test system

Fig. 4. Experimental apparatus. (a) Schematic of the performance test system (b) Photo of the performance test system.

Table 1
Dimensions of test problem.

Variable	Dimension(mm)
b_c	50
b_e	80
D_e	46
D_c	46
h	60
l_c	85
l_{ct}	212
l_e	100
l_{et}	285
r	12
R	80
X	70
X_c	40
X_e	80
Y	70

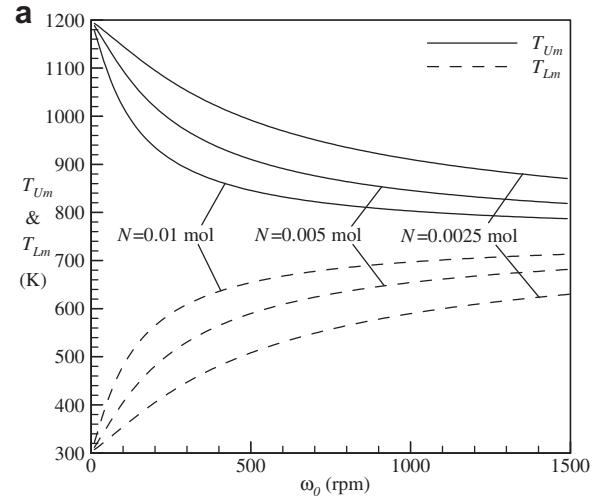
connected to a speed sensor, a torque sensor and a hysteresis brake. The hysteresis brake can exert different constant loadings to the engine. Once the loading is applied, the engine speed will experience a rapid decrease at the beginning, and then it reaches a stable regime. The engine torque and the corresponding operating speed were recorded after the stable regime is reached at different loadings. The shaft power output at different operating speed can be then obtained.

3. Results and discussion

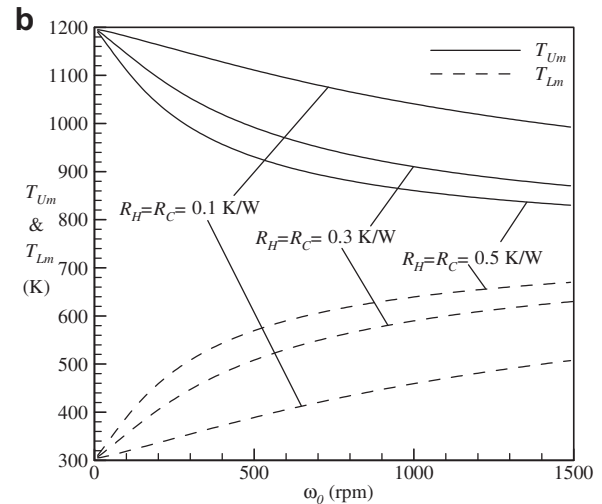
Fig. 5 shows the dependence of the mean fluid temperatures in the expansion and compression spaces (T_{UM} and T_{LM}) on the operating speed (ω_o), in rpm. In this figure, effects of molar number of air (N), thermal resistances of hot and cold sides (R_H and R_C) and effectivenesses of regenerator in heating and cooling processes (e_H and e_C) are displayed. It is found that as the operating speed of the engine is increased, the mean fluid temperature in the expansion space is decreased and that in the compression space is increased. In other words, the temperature difference between the expansion and the compression spaces is reduced as

Table 2
Engine specifications and operating parameters.

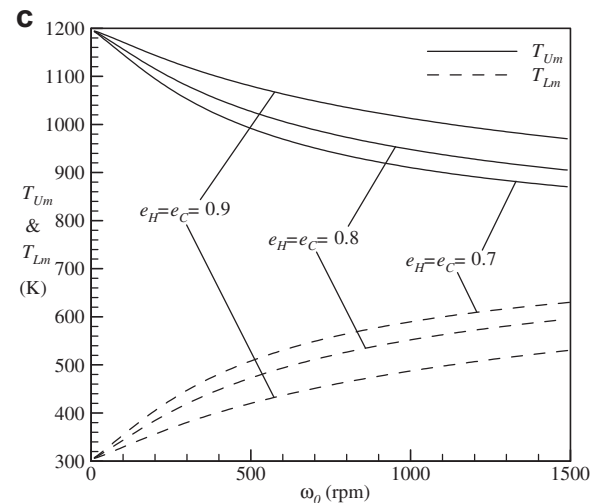
Working fluid	air
Charging pressure	1 atm
Displacer swept volume	82.85 cm ³
Piston swept volume	64.49 cm ³
Dead volume	24.61 cm ³
Compression ratio	3.37
Heating temperature, T_H	1200 K
Cooling temperature, T_C	300 K
Thermal resistance of hot side, R_H	0.82 K/W
Thermal resistance of cold side, R_C	0.61 K/W
Effectiveness of regenerator, $e_H = e_C$	0.69
The effectiveness of mechanism, E	0.7
Heating method	Infrared heater
Cooling method	Water jacket



Effects of molar number at $R_H = R_C = 0.3$ K/W and $e_H = e_C = 0.7$.



Effects of thermal resistances at $N = 0.0025$ mol and $e_H = e_C = 0.7$.



Effects of regenerator effectiveness at $N = 0.0025$ mol and $R_H = R_C = 0.3$ K/W.

Fig. 5. Mean fluid temperatures vs. operating speed. (a) Effects of molar number at $R_H = R_C = 0.3$ K/W and $e_H = e_C = 0.7$. (b) Effects of thermal resistances at $N = 0.0025$ mol and $e_H = e_C = 0.7$. (c) Effects of regenerator effectiveness at $N = 0.0025$ mol and $R_H = R_C = 0.3$ K/W.

the operating speed is increased. This is reasonable since at a higher speed, there is less time for the working fluid to absorb heat from the hot side thermal reservoir in the expansion space or to reject heat to the cold side thermal reservoir in the compression space. It is straightforward to expect that the indicated work output per cycle is decreased as the temperature difference is reduced.

In Fig. 5a, effects of the molar number of air at $R_H = R_C = 0.3$ K/W and $e_H = e_C = 0.7$ are evaluated. It is noted that as the mass of working fluid is increased, the difference between T_{UM} and T_{LM} is further reduced. This may be attributed to the increased thermal capacity when more air is charged. On the other hand, it is expected that when the thermal resistances are elevated or the regenerator effectiveness is decreased, the heat transfer is retarded so that the difference between T_{UM} and T_{LM} will be reduced. The results presented in Fig. 5b and c clearly reflect this expectation. The cases shown in Fig. 5b are fixed at $N = 0.0025$ mol and $e_H = e_C = 0.7$, and Fig. 5c at $N = 0.0025$ mol and $R_H = R_C = 0.3$ K/W.

Operating temperature ratio (τ) is calculated by Eq. (10) based on the mean temperatures in the expansion and the compression spaces obtained at different operating speeds, and the data are provided in Fig. 6 for the same cases considered in Fig. 5. The magnitude of the operating temperature ratio increases with the operating speed. In addition, it is observed that as the air mass is elevated, or the thermal resistances become higher, or the effectivenesses of the regenerator are lowered, the operating temperature ratio is increased.

A photo of the test engine is shown in Fig. 7. The variations of V_e , V_c , V and V_d with the crank angle (θ) are shown in Fig. 8. It is observed that the phase angle between the expansion and the compression spaces is 100° . The total volume is varied between 48 and 144 cm^3 , and the dead volume in the engine is 24 cm^3 . Numerical simulation of the test engine is carried out for the operating conditions at $e_H = e_C = 0.69$, $N = 0.0025$ mol, and $R_H = R_C = 0.3$ K/W.

Fig. 9 shows the predicted p - V diagrams of the test engine at different operating speeds. In this figure, it is clearly seen that an increase in the operating speed leads to a decrease in the area of the p - V diagram, which implies that the indicated work output (W) per cycle will be reduced as the engine speed is elevated. The reduction of the indicated work can be attributed to the decrease in the temperature difference between the expansion and the compression spaces as the operating speed is elevated, which is already observed in Figs. 5 and 6. Furthermore, the decrease in indicated work (W) per cycle due to an increase in the operating speed is accompanied by an increase in forced work (W_-). This can be illustrated in Fig. 3, in which the shaded area representing W_- is increased when the area of cycle shrinks. By Eq. (16), shaft work of engine W_s is calculated in terms of W and W_- . In this equation, it is found that as W is decreased and W_- is increased, the magnitude of W_s should be reduced obviously. Plotted in Fig. 10 is the influence of the operating speed on W , W_- , and W_s .

However, being aware that shaft power output is calculated in terms of shaft work output and operating speed by using Eq. (17), one should recognize that shaft power output tends to increase with operating speed. As a result, one may obtain a maximum shaft power output as the operating speed is varied in a certain range. Fig. 11 displays the dependence of shaft power output (P_s) of the test engine on the operating speed. Again, the effects of the molar number of air, thermal resistances of hot and cold sides and effectivenesses of regenerator in heating and cooling processes are evaluated. It is clearly seen that the shaft power output first increases with the operating speed in the low-speed regime. When the operating speed is elevated, the shaft power output reaches

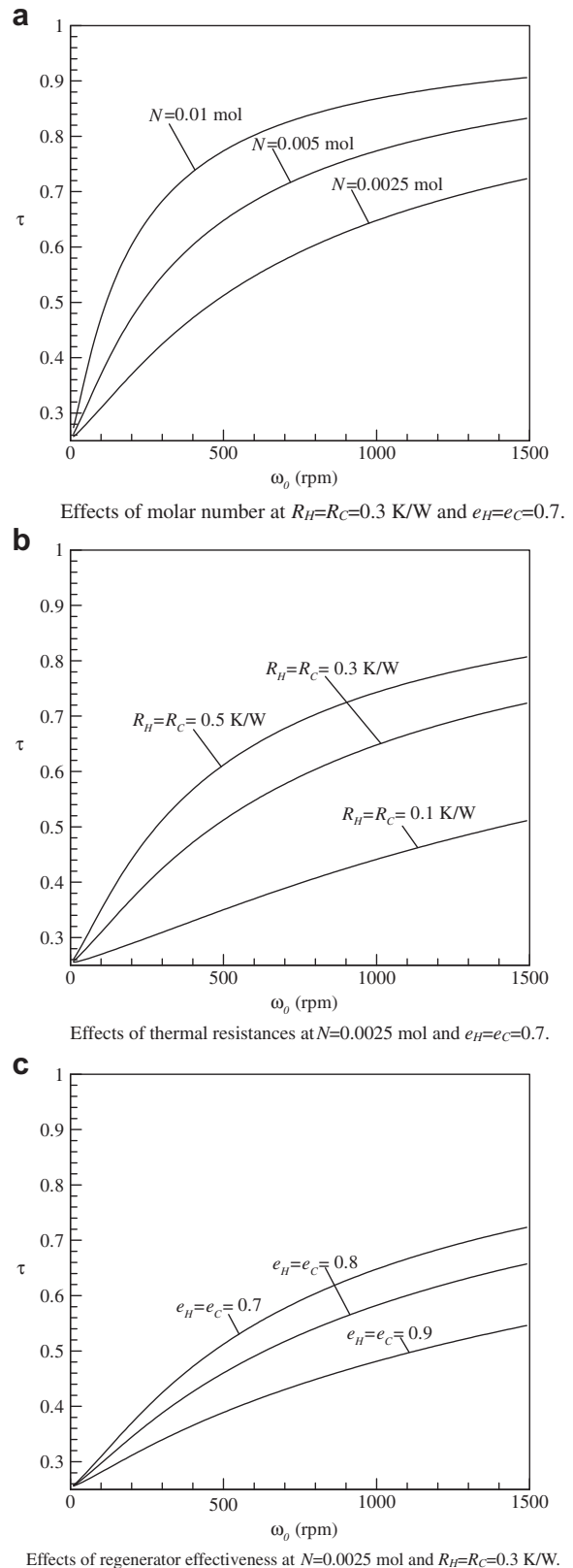


Fig. 6. Operating temperature ratio vs. operating speed. (a) Effects of molar number at $R_H = R_C = 0.3$ K/W and $e_H = e_C = 0.7$. (b) Effects of thermal resistances at $N = 0.0025$ mol and $e_H = e_C = 0.7$. (c) Effects of regenerator effectiveness at $N = 0.0025$ mol and $R_H = R_C = 0.3$ K/W.

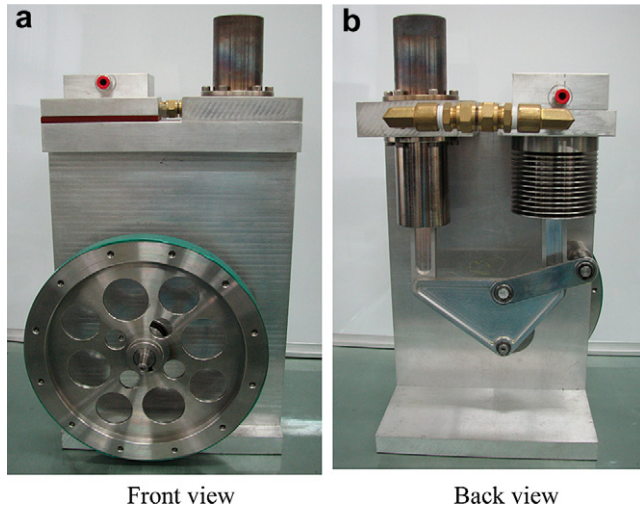


Fig. 7. Test engine.

a maximum at a critical operating speed. Over the critical operating speed, the shaft power output gradually descends in the high-speed regime. In addition, the peak shaft power output and its accompanying critical speed are strongly dependent on the molar number of the charged air, the thermal resistances, and the effectivenesses of the regenerator. In the plots of this figure, it is found that as the air mass is elevated, or the thermal resistances are reduced, or the effectivenesses of the regenerator are increased, the shaft power output can be remarkably increased. For example, in Fig. 11b, at $R_H = R_C = 0.5$ K/W the peak shaft power is around 6 W and the critical speed is 200 rpm. As the thermal resistances are reduced to be $R_H = R_C = 0.1$ K/W, the maximum shaft power is increased to 29 W and the critical speed shifts to 900 rpm.

It is important to mention that for the cases considered in Fig. 11a, the effect of the charged air mass is simply evaluated at fixed thermal resistances (0.3 K/W). As a matter of fact, the effect of the mass of charged air is rather involved. In general, when more air is charged into the working space of the engine, the thermal conductivity of the working fluid and the heat transfer coefficient on the walls of the hot and cold sides may be significantly increased

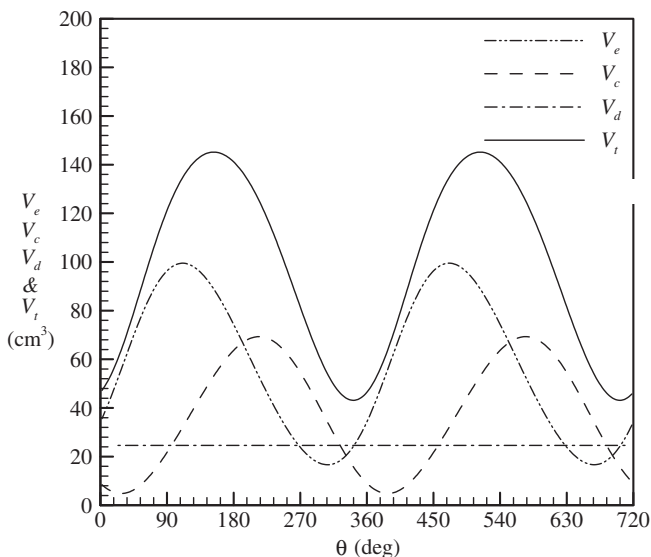
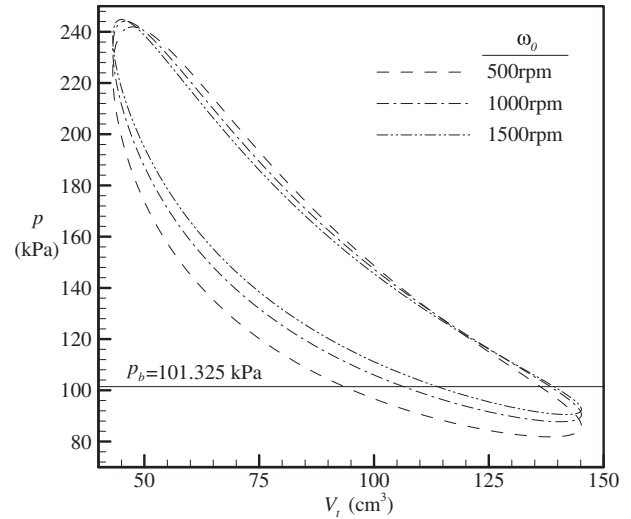


Fig. 8. Volumes' variation for the test engine.

Fig. 9. Predicted p - V diagrams at different operating speeds for the test engine.

[30,31] so that thermal resistances could be greatly reduced. As long as the related information is obtained, the information can be introduced into the present model to assess the effect of mass on the performance of the engine.

A comparison between the experimental and the numerical data for the shaft power output as a function of the operating speed is shown in Fig. 12a, and the relative error between these two sets of data is shown in Fig. 12b as a function of the engine speed. The range of the engine speed in the experiments is within 130 and 400 rpm, and the maximum shaft power output is found to be 4.4 W at 200 rpm. The experimental results closely agree with the numerical predictions at high-speed regime. Generally, the relative error is within 30%. However, in Fig. 12a, the absolute error between the experimental and numerical results is remarkable when the engine speed is lower than 200 rpm. This may probably be attributed to the greater uncertainty of the measurement data at low engine speeds. For the test engine, the engine cannot be operated at a speed lower than 130 rpm.

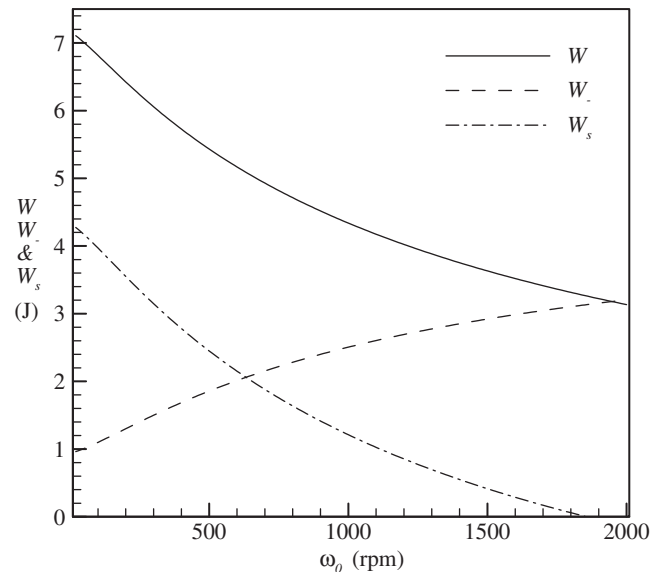


Fig. 10. Predicted effects of operating speed on indicated work, forced work, and shaft work for the test engine.

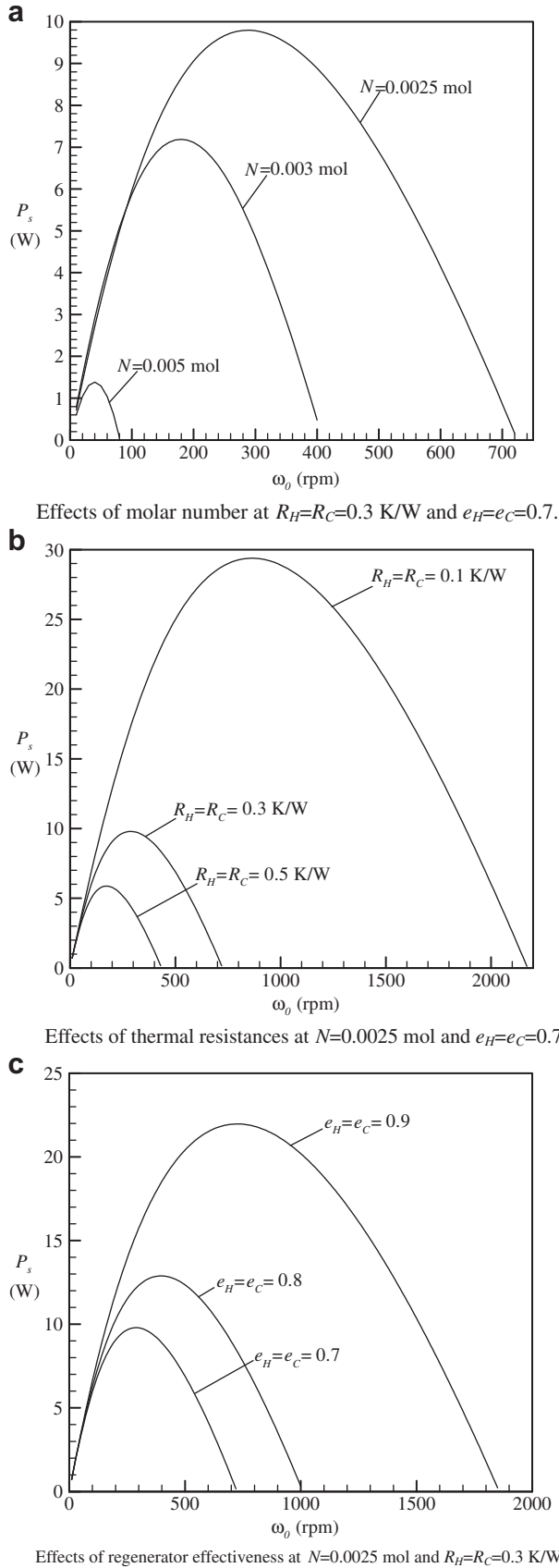


Fig. 11. Predicted shaft power output vs. operating speed for the test engine. (a) Effects of molar number at $R_H = R_C = 0.3$ K/W and $e_H = e_C = 0.7$. (b) Effects of thermal resistances at $N = 0.0025$ mol and $e_H = e_C = 0.7$. (c) Effects of regenerator effectiveness at $N = 0.0025$ mol and $R_H = R_C = 0.3$ K/W.

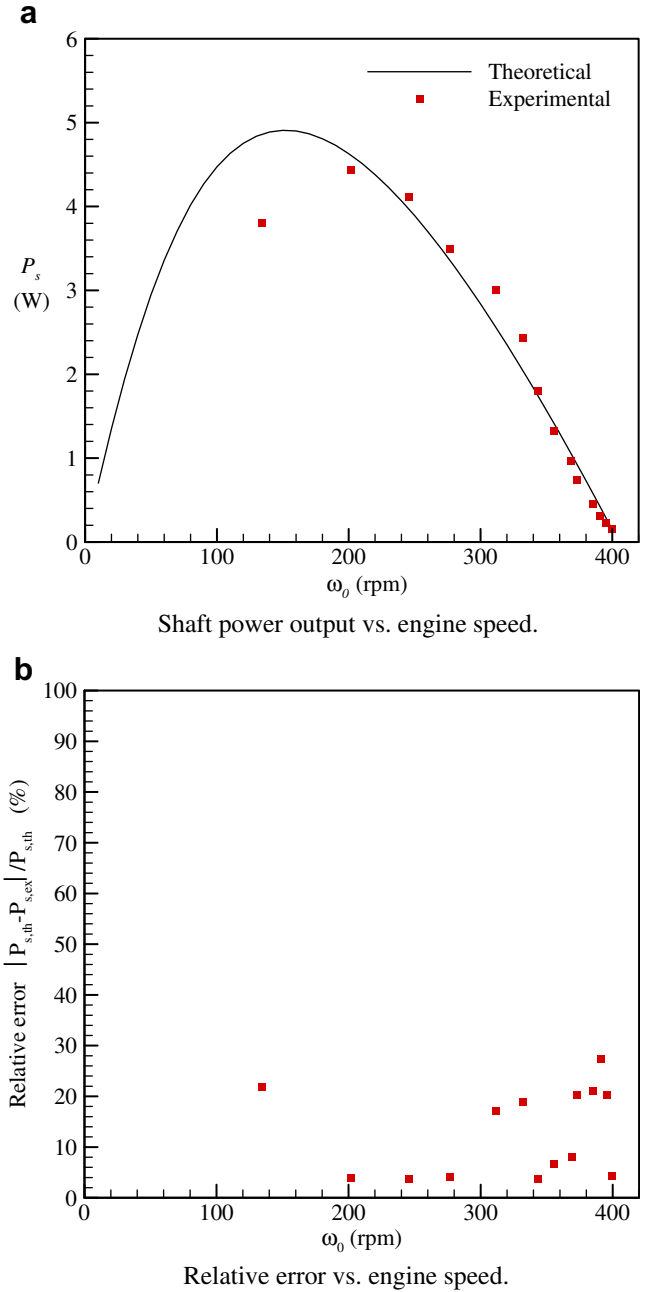


Fig. 12. Comparison between experimental and numerical data. (a) Shaft power output vs. engine speed. (b) Relative error vs. engine speed.

4. Conclusions

In this study, an efficient lumped-mass model has been developed and used to predict the relationship between the operating speed and shaft power output of the Stirling engines. Temperature variations in the expansion and the compression spaces as well as the shaft power output corresponding to different operating speeds are of major concerns. The concept of the mechanism effectiveness initiated by Senft [15,29] is introduced in this study to calculate the shaft work, in terms of the indicated work, the forced work, and the mechanism effectiveness. The effects of the operating parameters on the power output of the engine are also evaluated.

Results show that as the operating speed is increased, the temperature difference between the expansion and the compression spaces is reduced and as a result, the shaft work

output decreased. However, the shaft power output is determined in terms of the shaft work output and the operating speed. When the operating speed is elevated, the shaft power output reaches its maximum at a critical operating speed. Over the critical operating speed, the shaft power output descends in the high-speed regime. In addition, the peak shaft power output and its accompanying critical speed are strongly dependent on the molar number of charged air, the thermal resistances, and the effectivenesses of regenerator. As the air mass is decreased, or the thermal resistances are lowered, or the effectivenesses of the regenerator are elevated, the power of the engine and the critical speed are both elevated.

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